Millimeter Observing Techniques: Calibration and Imaging of Spectral Line Data

Bertrand Lefloch (LAOG, France)
Outline of the Lecture:

0. Introduction
1. Calibration
   1.1 Beam and Antenna Properties
   1.2 Atmosphere
   1.3 Receiver Calibration
2. Heterodyne Detection
3. Spectral Line Observations: spectral surveys and mapping
Summary of Lectures 1 – 2

Atmosphere

Lecture 3

\[ T'_A(\theta, \phi) = \frac{1}{\Omega_A} \int_{\text{source}} P(\theta - \theta', \phi - \phi') J_\nu(T_B) \psi(\theta', \phi') d\Omega' \]

- corrected for atmospheric absorption
- Telescope Beam
- Source Brightness Distribution
Heterodyne Detection
The antenna is the transition between the free space propagating wave and the guided wave in the feedhorn but the reflector (antenna dish) is separated from the focal antenna. Cassegrain antennas are widely used because of the wide fov in the image focal plane that allows the simultaneous use of several receivers with a weak gain loss and beam degradation for the out of axis receivers.

Focal antennas are usually feedhorns that collect the signal to a waveguide, which focuses the RF signal and the LO signal to a mixer. A horn pattern is well suited to illuminate the secondary reflector of the telescope. At high ν (> 400 GHz), technological limitations lead to prefer other devices, such as open structures (integrated planar antennas, etc...). Focal antennas are used to optimize the coupling between the mixer and the telescope by maximizing the illumination and minimizing the spillover. The antenna must then radiate a circular and symmetrical pattern with weak sidelobes. It must also have negligible ohmic and dielectric losses. Last, the beam must be well coupled to the fundamental gaussian mode of the waveguide.
Amplifier: Low noise amplifiers are needed to increase the power of the astronomical signal before being able to analyze the signal. FET AsGa transistors have been widely used as their working bandwidth can be as broad as 10 Ghz. High Electronic Mobility Transistors (HEMTS) have typical noise temperatures of a few K/GHz. These amplifiers are currently used for heterodyne receivers at millimeter wavelengths.

Mixer is the key element of the heterodyne receiver. It controls the interaction of the incoming signal with the reference signal from the LO, and converts the signal at radio frequency to intermediate frequency. Its dimensions must be of the order of the wavelength for efficient absorption/detection of photons. The transmission of signals must be carried out with well matched waveguides. The mixer performance is characterized by the I – V curve, which deviates from linearity at the operating point. Devices currently used today are SIS (Superconducting – Insulator – Superconducting) mixers.

Local Oscillator: provides a reference to the mixer. The power delivered depends on the kind of mixer: typically 10nW at 200 GHz for an SIS junction. Gunn diodes are currently used. Other types of devices (Impatt diodes, quantum well devices) have appeared in the past years (Gronqvist, Rydberg, Proc. Eur. Microwave Conf. 1988) which can provide a power of a few tens of μW at millimeter wavelengths.

Beyond 120GHz, the LO is combined with a frequency multiplier (2X,3X). In the submm domain, lasers are also used as LO (very stable).
Mixer is the key element of the heterodyne receiver:
- It controls the interaction of the incoming RF signal (100 GHz) with the reference signal from the LO to convert it to IF signal (< a few GHz), to be processed and analyzed by spectrometers.
- The mixer performance is determined by a non-linear I–V relation.

Several techniques used (e.g. Schottky). The most commonly used devices are SIS junctions: two superconducting layers separated by a thin insulator. Once the junction is polarized (V_{DC}) , absorption of photons (LO and RF signal) help e− tunnel through the junction (jump to the conduction band in the other superconducting layer → e− current → highly non-linear function of voltage applied.

(see Tucker & Feldman, 1985, IEEE journ. of Quantum Electronics, QE-15, No11)
SIS Mixer

Bias voltage $V_0 < V_{\text{gap}}$

Quasiparticles of one supraconductor can absorb impinging photons

Photon assisted Tunneling

The maximum frequency is given by $V_{\text{gap}} \approx 1\text{mV} \approx 241\text{GHz}$

Modern material (NbN, NbTiN) allow operation up to 1.2 THz

Maximum working freq. = $V_{\text{gap}}$
Coherent Detection: I–V Curve

LO and RF signal cause voltage fluctuations $\delta U$ at the working point $(I_{DC}, U_{DC})$

$I(U + \delta U) = I_0 + a_1 \delta U + a_2 \delta U^2 + \ldots$  
and $\delta U^2 = \left\{ E \sin(\omega_s t + \delta_s) + V \sin(\omega_L t + \delta_L) \right\}^2$

$\delta U^2 = E^2 \sin^2 (\omega_s t + \delta_s) + V^2 \sin^2 (\omega_L t + \delta_L) + 2E.V.\sin(\omega_s t + \delta_s) .\sin(\omega_L t + \delta_L) - \cos(2 .(\omega_s t + \delta_s)) / 2$

$I = I_0 + a_2 (E^2 + V^2) / 2$

DC

$+ a_1 E \sin(\omega_s t + \delta_s)$  
$1$st harm

$+ a_1 V \sin(\omega_L t + \delta_L)$  
$1$st harm

$- a_2 E^2 \cos(2 \omega_s t + 2\delta_s) / 2$  
$2$nd harm.

$- a_2 V^2 \cos(2 \omega_L t + 2\delta_L) / 2$  
$2$nd harm. of LO

$+ a_2 E.V. \cos((\omega_s - \omega_L)t + \delta_s - \delta_L)$  
$\leftarrow V_{IF}$

$- a_2 E.V.\cos((\omega_s + \omega_L)t + \delta_s + \delta_L)$

Translation $E(\omega_s) \rightarrow E(\omega_{IF})$

The last two terms depend only on the first power of the signal and the LO → the amplitude is a fair reproduction of the input RF signal. The output power is linearly dependent on the LO power. By increasing the LO power, the gain of the total system is increased (usually the LO power is much larger than the signal power).

Any variation of the LO power will appear as a variation of the total gain: Output power stability of the LO is necessary...
Coherent Detection

- By using an appropriate filter, all unwanted signals can be suppressed → the mixer can be considered as a linear device producing an output signal at frequency $\omega_{\text{IF}} = \omega_s - \omega_L$

Two radio frequency signals are converted to $\omega_{\text{IF}}$, corresponding to $| \omega_L - \omega_s | = \omega_{\text{IF}}$:
- $\omega_s = \omega_L + \omega_{\text{IF}} \quad \text{USB}$ and $\omega_s = \omega_L - \omega_{\text{IF}} \quad \text{LSB}$

A mixer will transform these 2 bands into the same IF; the 2 bands are placed symmetrically with respect to $\omega_L$: mirror frequencies. They are detected superposed in the same IF band.

How to identify, i.e. discriminate between lines from the Upper and Lower bands? Tuning at a slightly higher freq. → higher $\omega_L$

Upper band: at the left edge of the IF band are found the lowest freq.: 
- lines appear to shift towards the left.

Lower band: at the right edge are found the lowest freq., which disappear from the IF band. 
- lines appear to shift towards the right.
Three types of Heterodyne Receivers

- **DSB** receiver: both mirror frequencies are present. This is used mostly in the submm domain. Both sidebands can be recovered by a deconvolution algorithm in principle.

- **SSB** receiver: one of the mirror frequencies is *rejected*, either the lower (→ USB) or upper (→ LSB). Elimination of one band is achieved by placing a Sideband filter: Fabry–Perot, a narrow bandpass, tuning. Signal and Image band gains: $G_s, G_i$ \quad \left( G_s + G_i = 1 \right) \quad \text{image to signal band ratio: } G_{im} = G_i / G_s$

- **2SB** receiver: mixers separating sidebands → EMIR90 and E330
Signal Processing

RF signal

IF signal: spectral distribution
$S(\nu)$ is translated to $\nu_{\text{IF}}$

Sq. Law Detector

Integration

Collected signal

IRAM Summer School 2009
Backends

Output of the Receiver $V(t)$

We look for $S(\nu)$

Two types of backends

- Analogic: filterbanks, Acousto–Optical Spectrometers
easy to build, expensive, no flexibility (resolution and bandwidth)

- Digital: correlators, FTS
  Adjustable resolution and bandwidth
Digital Backends

FTS (available soon at IRAM 30m)

Theorem of Wiener–Kinchin

Autocorrelator (VESPA, WILMA at IRAM 30m)

Fig. 4.1. A sketch of the relation between the voltage input as a function of time, $V(t)$, and frequency, $V(\nu)$, with the autocorrelation function function, ACF, $R(\tau)$ and corresponding power spectral density, PSD, $S(\nu)$. The two-headed arrow represents reversible processes.
**Receiver Stability**

The signal measured at the output of the receiver is:

\[ W = k (T_{sys} + T_a) G \Delta v \]

So,

\[ \frac{\Delta T}{T_{sys}} = \frac{\Delta G}{G} \]

*Gain fluctuations contribute to limiting the sensitivity...*

It is a priori not possible to distinguish temperature and gain fluctuations. The gain must be extremely stable to be able to measure a small variation of the antenna temperature (the astrophysical signal). This is achieved by using a system based on a differential compensation principle.

The receiver is switched regularly between the antenna and a resistive load in equilibrium at temperature Tref. The difference of the 2 signals is measured using a phase-sensitive detector:

\[ W_a - W_r = k(T_a - T_{ref}) G \Delta f \]

Now:

\[ k(T_a - T_{ref}) (G + \Delta G) \Delta f = k(T_a + \Delta T_a - T_{ref}) G \Delta f \rightarrow \frac{\Delta T_a}{T_{sys}} = \frac{(\Delta G/G) ((T_a - T_{ref})/T_{sys})}{(T_a - T_{ref})/T_{sys}} \]

A gain fluctuation will have a reduced influence depending on Ta–Tref, or even null, if the receiver is balanced.

There are several methods to produce the comparison load Tref: a mean value of the sky temperature obtained by beam switching, which allows to eliminate part of irregularities caused by the earth’s atmosphere.

Tsys contains only the electronic chain.
Receiver Noise and Detectability Limit

The total power of the system is split into: Antenna (atmosphere, spillover losses,..) and Receiver (+backend): thermal noise, loss in transmission,...

\[ P_{sys} = P_A + P_{rec}, \quad \text{or} \quad T_{sys} = T_A + T_R \]

- The detection limit of a receiver is given by the power fluctuations of the system, which can be expressed as:

\[ \Delta T_{rms} = \frac{T_{sys}}{(\Delta v \cdot \tau)^{1/2}} \]

- \( \Delta v = \) bandwidth in which the signal is measured
- \( \tau = \) integration time ON source

<table>
<thead>
<tr>
<th>Time</th>
<th>1 sec</th>
<th>1 hr</th>
<th>16 hrs</th>
<th>64 hrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\Delta T_{rms}}{T_{sys}} )</td>
<td>( \frac{1}{(\Delta v)^{1/2}} )</td>
<td>( \frac{0.016}{(\Delta v)^{1/2}} )</td>
<td>( \frac{0.004}{(\Delta v)^{1/2}} )</td>
<td>( \frac{0.002}{(\Delta v)^{1/2}} )</td>
</tr>
</tbody>
</table>

Broad band measurements: keep \( T_{sys} \) small! Good to have \( \Delta v \) large (bolometers) or new Rx generations (ALMA, PdBI,etc..)

Narrow spectral lines: coherent receivers can have \( \Delta v \) (almost) as small as you want! \( \Delta v \) is determined by the kinematics of the source: 1MHz \( \approx 1 \) km/s @ 1mm, and is given by the Doppler relation: \( \Delta v / c = \Delta v / \nu \)

For a 1/100 SNR in 1 sec: \( \approx 1/1 \) in 1 hr, \( \approx 2.5 \) to 1 in 16 hrs;

An rms of \( (1 \sigma) \) of \( \approx 10 \) mK is reached in 1 hr of integration time in a bandwidth of 1 Mhz for \( T_{sys} = 100K \).
Receiver stability and sensitivity

- Some fraction of the time is used to control to keep the gain constant, and a Dicke receiver uses half of the integration time to measure $T_{\text{ref}}$. Moreover, both signals $T_a$ and $T_{\text{ref}}$ have similar errors (both containing an atm contribution) so that the actual sensitivity is worse by a factor of 2:

$$\Delta T_{rms} = \frac{K T_{\text{sys}}}{\Delta \nu \tau}$$

Here $\tau$ is the integration time.

- Wobbler switching or beam switching mode: $K \sim 2$
- Freq.-switching, the sensitivity is higher by a factor $\sqrt{2}$.

But... integration time is not the telescope time!

- Dead times between phases
- Time between subscans (backend sync.)
- Telescope motion
- + receiver tuning, pointing, focussing,
  (computer crash....)

Use the IRAM Time Estimator
Time Estimator

WSW Observations

Radiometer formula: 5.4 mK
Measured: 6.2 mK
Minimum Receiver Noise Temperature

The intrinsic limitation on $T_R$ comes from the Heisenberg relations:

$$\Delta E \Delta t \geq \hbar/4\pi$$

Or

$$\Delta n \Delta \Phi \geq 1/2$$

$$T_{\text{min}} = \frac{h \nu}{k} = 11\text{K at } 230\text{GHz}$$

Quantum limit!

EMIR receivers:
- $T_r = 25–50\text{ K} = \text{a few } T_{\text{min}}$
- stable
- Broad IF: 4GHz (8GHz at 3mm)
Heterodyne Instrumentation at the IRAM 30m
EMIR Receivers

Four single pixel receivers

<table>
<thead>
<tr>
<th>EMIR band</th>
<th>$F_{sky}$ (GHz)</th>
<th>mixer type</th>
<th>polarization</th>
<th>IF width (GHz)</th>
<th>$T_{sb}$ (K)</th>
<th>$G_{im}$ (dB)</th>
<th>combinations</th>
<th>$T_{db}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E 090</td>
<td>83 – 117</td>
<td>2SB</td>
<td>H/V</td>
<td>8</td>
<td>50</td>
<td>&gt; 13</td>
<td>X</td>
<td>65</td>
</tr>
<tr>
<td>E 150</td>
<td>129 – 174</td>
<td>SSB</td>
<td>H/V</td>
<td>4</td>
<td>50</td>
<td>&gt; 10</td>
<td>X</td>
<td>65</td>
</tr>
<tr>
<td>E 230</td>
<td>200 – 267</td>
<td>SSB</td>
<td>H/V</td>
<td>4</td>
<td>50</td>
<td>&gt; 13</td>
<td>X</td>
<td>65</td>
</tr>
<tr>
<td>E 330</td>
<td>260 – 360</td>
<td>2SB</td>
<td>H/V</td>
<td>4</td>
<td>70</td>
<td>&gt; 10</td>
<td>X</td>
<td>85</td>
</tr>
</tbody>
</table>

A large and unique flexibility in Rx combinations

Observations down to 76 GHz are possible

(C. Kramer)
EMIR Receivers

16 GHz of instantaneous bandwidth!
HERA

**Incompatible with EMIR**

**HERA: 18 pixel array**

- 3x3 pattern, sep. of 24”
- 2 polarizations: 18 pixels
- IF: up to 1GHz (with 2–4MHz res.)
- HPBW: 9” – 12”

210–276 GHz: Pol 1
210–242 GHz: Pol 2

Derotator K–mirror

Pixels of the array as seen in the Rx framework ≠ Orientation on the sky!

The orientation of the sky is controlled by the K–mirror (derotator) → 2 mapping modes.
HERA: the IRAM–30m multibeam

Efficient Mapping Mode

→ Slightly undersampled map in one shift

Coarse or oversampling mode

Very Good Baseline stability in FSW
Backends at the IRAM 30m Telescope

Up to 16 GHz of bandwidth available instantaneously!

4MHz filterbanks: 2 bandwidths of 4 Ghz each at 4 MHz resolution
→ 2 EMIR Rx

WILMA autocorrelator: 16 bands of 1GHz at 2 MHz res.
→ 4 x 4GHz (2 EMIR Rx in 2 pol.)

VESPA: narrow band autocorrelator: 4 x 640 MHz available, in the center of the IF bands.
High-flexibility in resolution: 3.3 kHz – 1.25 MHz
18000 channels available in the following resolution/bandwidth mode: 20 kHz/40 MHz – 40 kHz/80MHz – 80 kHz/160 MHz – 320 kHz/320 MHz – 1250 kHz/640 MHz

PS: Carsten, we all wait for the new FTS!
Yes, they can!

2 setups in 30min to cover 32 GHz at 3mm!

2x 8 Ghz LSB & USB covered with WILMA
Spectral Line Observations
Why Switching?

Because the atmosphere is inhomogeneous and time-varying...

\[ T_{\text{sys}} = T_A + T_{\text{sky}} + T_{\text{spillover}} + T_R \]

\[ T_A \ll T_{\text{sys}} \text{ most of the time} \]

\[ T_A \sim 1 \text{ K} \]
\[ T_{\text{sky}} \sim 30\text{K (3mm)} \]
\[ T_{\text{spillover}} \sim 20\text{K} \]
\[ T_R \sim 50\text{K} \]

Two enemies:
- Sky emissivity fluctuations
- Receiver gain fluctuations

Solutions for single px receivers:

Most of the time:
- Position switching
- Wobbler switching
- Frequency switching

Some of the time:
- On-the-fly (fast scanning)
Performing an integration On source followed by an integration Off source at a fixed position on the sky:

$$T_A + T_{\text{sky}} + T_{\text{spillover}} + T_R$$

It is important to calibrate the atmosphere often enough, depending on the stability of the receiver and the atmosphere.

The duration of a data acquisition will be determined also by the stability of the receiver and the atmosphere.

Choice of OFF position: should be $\approx$ at the same airmass, closeby (fast to move and come back), emission free (sometimes difficult to achieve...).

Large offsets are possible but may affect baseline stability.
WSwitching

This is a variant of Position Switching in which the secondary mirror nutates, up to a few Hz, instead of moving the telescope.

Spectroscopic baseline is affected by gain fluctuations and by reflections inside the telescope.

**wobbl er switching with period of a few sec** cancels out most atmospheric fluctuations.

Telescope nodding : removes standing waves

Telescope anti-nodding (sym. Mode) : removes slow linear drifts

**Drawbacks :**

Throw limited to +/- 2 arcmin  →  works only for compact sources. The Off position rotates on the sky (beware !).

Observations are made off-axis, which may induce some optical aberrations (coma) at high frequency when large throws are used.
Fswitching

The reference is frequency, not space. The frequency of the LO changes regularly by a small amount, which produces a comparison signal.

Very efficient as all the time is spent on source.

spares search for emission–free reference position.

Baseline can become quite ugly… and messy for complex spectra. Only reasonable for narrow lines (dark clouds).

Beware of CO mesospheric lines
Baseline Ripple

A system of standing waves develops between reflecting surfaces in the telescope (mirrors, ...)

The feed horn does not absorb all the incoming power.

Best known ripples

7.5 MHz between subreflector and Rx
15.5 MHz between vertex and Rx
The instrumentation of the IRAM 30m telescope is remarkably efficient to carry out spectral surveys and spectral line mapping:

so what?

- Spectral line mapping carries information on spatial and velocity distributions: *needed for all projects where dynamics are essential...*
- Extragalactic studies: galactic, structure, dynamical evolution
- Cloud structure, ISM: turbulence, core formation
- Star Formation: various scenarios of gravitational collapse exist, which have different kinematical signatures, from cloud scale (1–10 pc) → core (0.1 pc, 2x10^4 AU) → protostellar envelope (10^3 AU)
  - (protostellar disk ...)
  - at the distance of Taurus (140 pc): 0.4 deg → 140” → 10”

...but not only!
Spectral Line Mapping

Dame et al. (2001)  resolution: 510''

Goldsmith et al. (2008)  resolution: 45''

Lefloch et al. (2009)
resolution 11''
1 sq deg
3.5x10^5 spectra

100 sq deg
3x10^6 spectra
20'' sampling
Introduction: Why spectral surveys?

Spectral surveys aim at exploring the chemical composition of interstellar matter, and its evolution along with stellar and Galactic evolution.

- Inventory of the gas coolants
- Census of the chemical composition
- Search for/identify new molecular species.
- Reconstruct the structure of the emitting source from analysis of multiple line profiles probing regions of different excitation conditions (very challenging!)

\[
\frac{3k}{8\pi^3} \frac{T_{mb} dV}{\mu^2 v S} = N_u = \frac{N_T}{Q(T_{ROT})} e^{-E_u/kT_{ROT}}
\]
Chemistry and Dynamics are closely coupled in many cases.

Star Formation studies:
In protostellar envelopes, the ionization fraction $x$ controls coupling with $B$ → gravitational collapse → compression heating of the envelope, $T(r)$ → ionization fraction (chemical network)...

Your astrophysical pet is *not* a spherical cow! A comprehensive study requires more than one pointed observation as chemical and physical conditions are not homogeneous.

- Multi-transitions maps are sometimes necessary to analyze the physical conditions.

Determination of the Physical and Chemical conditions is an iterative process

- Obs. ← Line Predictions
  - Source model $n(r), T(r), v(r)$
    - Rad. Transfer code
    - Chem. network
  - X(r)
    - Rad. Transfer code
Example II: Unbiased Spectral Survey of L1157-B1 with HIFI/HERSCHEL

The archetype of chemically active outflows

Benedettini et al. (2007)

Multiple components. Different species peak in different parts of the walls of the cavity.

See talk by E. Caux

CH$_3$OH ($2_k$-$1_k$) PACS

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Spectral Line Mapping: Why heterodyne arrays?

- A dense core: 0.1 pc → 140” at 140pc, need HPBW (resolution) < 70” to spatially resolve the core.

- Cloud/filament: 10pc: 4 deg at 140pc ≈ 14000”
  IRAM 30m telescope @ 230GHz: HPBW = 11” → ≈1300 pointed obs.

- Current SIS receivers are getting closer to the quantum limit: to increase the efficiency, an easy solution: increase the number of Rx: 1 → N_{pix}

- Beware: each Rx must be excellent; they should all have similar performances:
  \[ T_{int} \propto T_{sys}^2 / N_{pix} \]
## Spectral Line Mapping: 15 yrs of progress at IRAM

<table>
<thead>
<tr>
<th>Obs. Mode</th>
<th>Switching Mode</th>
<th>Nrec</th>
</tr>
</thead>
<tbody>
<tr>
<td>– 1996</td>
<td>raster</td>
<td>Psw / Fsw, 1 x 4</td>
</tr>
<tr>
<td>1996 – 2002</td>
<td>On The Fly</td>
<td>Psw, 1 x 4</td>
</tr>
<tr>
<td>2002</td>
<td>On The Fly</td>
<td>Psw, Fsw, 9 x 1 (HERA 1)</td>
</tr>
<tr>
<td>2003</td>
<td>OTF</td>
<td>Psw, Fsw, 9 x 2 (HERA 2)</td>
</tr>
<tr>
<td>2009</td>
<td>OTF</td>
<td>Psw, Fsw, 1 x 2 (EMIR)</td>
</tr>
<tr>
<td>?</td>
<td>OTF</td>
<td>Psw, Fsw, S–HERA ?</td>
</tr>
</tbody>
</table>

*There is no heterodyne array at 3mm at the moment.*
## Spectral Line Mapping: Arrays

<table>
<thead>
<tr>
<th>Telescope</th>
<th>$D$ m</th>
<th>Rx</th>
<th>config</th>
<th>freq. GHz</th>
<th>spacing ''</th>
<th>HPBW ''</th>
<th>$D/\lambda \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRAM-30m</td>
<td>30</td>
<td>HERA</td>
<td>$2(3 \times 3)$</td>
<td>220–270</td>
<td>24</td>
<td>9–11</td>
<td>27</td>
</tr>
<tr>
<td>JCMT</td>
<td>15</td>
<td>HARP</td>
<td>$4 \times 4$</td>
<td>325–375</td>
<td>30</td>
<td>14</td>
<td>17</td>
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<tr>
<td>Nobeyama</td>
<td>45</td>
<td>BEARS</td>
<td>$5 \times 5$</td>
<td>82–116</td>
<td>41.1</td>
<td>20–15</td>
<td>12</td>
</tr>
<tr>
<td>NRAO</td>
<td>12</td>
<td></td>
<td>$2 \times 4$</td>
<td>220–250</td>
<td>87</td>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>NANTEN2</td>
<td>4</td>
<td>SMART</td>
<td>$4 \times 2$</td>
<td>490–810</td>
<td>40–22</td>
<td>10</td>
<td></td>
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<tr>
<td>Parkes</td>
<td>64</td>
<td></td>
<td>13</td>
<td>1.4</td>
<td>1720</td>
<td>860</td>
<td>0.3</td>
</tr>
<tr>
<td>‡FCRAO</td>
<td>14</td>
<td>SEQUOIA</td>
<td>$2(4 \times 4)$</td>
<td>85–115.6</td>
<td>24</td>
<td>45</td>
<td>5</td>
</tr>
<tr>
<td>LMT</td>
<td>50</td>
<td>SEQUOIA</td>
<td>$2(4 \times 4)$</td>
<td>85–115.6</td>
<td>88</td>
<td>14</td>
<td>19</td>
</tr>
</tbody>
</table>
Raster Mapping

- integration while the telescope is tracking a fixed position, during $t_{ON}$
- then move to next position
- sky sampling is $\theta_s$
- go to OFF position, $t_{OFF}$
- do a calibration every $t_{CAL}$
Raster Mapping: Time Estimate

- a row with \( n \) pointings, sharing the same OFF position with
  \[ t_{\text{OFF}} = \alpha t_{\text{ON}} : \alpha = ? \]

- \( t_{\text{row}} = N t_{\text{ON}} + t_{\text{OFF}} \) so \( t_{\text{spec}} = t_{\text{row}} / n = t_{\text{ON}} (1 + \alpha / n) \)

- \( t_{\text{ON}} \) from the radiometer equation: \( \sigma_{\text{ON}} = \frac{\kappa T_{\text{sys}}}{\sqrt{B t_{\text{ON}}}} \), \( \kappa \approx \sqrt{2} \)

- \( \sigma_{\text{spec}} = \sqrt{\sigma_{\text{ON}}^2 + \sigma_{\text{OFF}}^2} = \sigma_{\text{ON}} \sqrt{1 + t_{\text{ON}} / t_{\text{OFF}}} = \sigma_{\text{ON}} \sqrt{1 + 1 / \alpha} \)

- therefore: \( \sigma_{\text{spec}} = \frac{\kappa T_{\text{sys}}}{\sqrt{B t_{\text{ON}}}} \sqrt{1 + 1 / \alpha} \)

- which we rewrite: \( \sigma_{\text{spec}} = \frac{\kappa T_{\text{sys}}}{\sqrt{B t_{\text{spec}}}} (1 + 1 / \alpha)^{1/2} (1 + \alpha / n)^{1/2} \)

- \( \frac{\partial \sigma_{\text{spec}}}{\partial \alpha} = 0 \Rightarrow \alpha = \sqrt{n} : \text{optimal choice for } \alpha \)

\[ t_{\text{OFF}} = \sqrt{nt_{\text{ON}}} \]
On-The-Fly (OTF) Mapping with reference position

- the telescope drifts across the map at a velocity $v_{\text{scan}}$ (in ”/s)
- data are dumped every $t_{\text{dump}}$ along each row
- the sky sampling along the scanning direction is $\theta_s = v_{\text{scan}} \cdot t_{\text{dump}}$
- the sky sampling perpendicular to the scanning direction is $\theta_{\text{row}}$
- integrate on OFF position for $t_{\text{OFF}}$
- do a calibration every $t_{\text{CAL}}$
OTF Mapping with reference position: Time Estimate

- number of dumps between two OFF measurements: $N$
- here again, optimal choice: $t_{OFF} = \sqrt{N} t_{dump}$
- but, in OTF:
  - $\theta_s$ smaller than in raster mapping
  - $t_{dump} \ll t_{ON}$

$\Rightarrow$ likely to average several dumps

- dumps are not independant and if $\sigma_{OFF}$ is not $\ll \sigma_{dump}$, the noise in the average dump will not decrease as Poisson noise
- you must ensure that $\sigma^2_{OFF} \ll \sigma^2_{dump}$
  - $\alpha > 10$ even if $N < 100$
- usually, $t_{dump} \approx 1$ s, so it is easy to guarantee $t_{OFF} > 10 t_{dump}$ ! (so please do)
OTF Mapping without reference position

- The telescope drifts across the map at a velocity $v_{\text{scan}}$ (in $\arcsec/s$).
- Data are dumped every $t_{\text{phase}}$ along each row.
- The sky sampling along the scanning direction is
  \[ \theta_s = v_{\text{scan}} \cdot t_{\text{dump}} \]
- The sky sampling perpendicular to the scanning direction is $\theta_{\text{row}}$.
- No integration on OFF position.
- 100% time ON the source.
- Do a calibration every $t_{\text{CAL}}$. 
OTF with HERA: undersampled mapping
OTF with HERA: oversampled mapping
Time Estimate: General Case

- $L_x, L_y, \sigma_{\text{pixel}}, \theta_{\text{pixel}}, \sigma_{\text{dump}}, \theta_{\text{dump}}, \delta \nu, \nu_{\text{scan}}, N_{\text{pix}}$
- $t_{\text{OTF}} = \eta_{\text{over}} N_{\text{row}} t_{\text{row}} = \eta_{\text{over}} \frac{L_y}{\theta_{mb}/\eta_{\text{samp}}} N_{\text{dump}} t_{\text{dump}} (1 + 1/\sqrt{N_{\text{dump}}})$
- Scanning velocity: $t_{\text{dump}} = \theta_{\text{dump}}/\nu_{\text{scan}}, N_{\text{dump}} t_{\text{dump}} = L_x/\nu_{\text{scan}}$
- RMS noise in final pixel: $\sigma_{\text{pixel}} = \sigma_{\text{dump}} \left( \frac{\theta_{\text{dump}}}{\theta_{\text{pixel}}} \right)^{1/2}$, 
  $\sigma_{\text{pixel}} = \frac{\kappa \sqrt{2} T_{\text{sys}}}{\sqrt{\delta \nu} t_{\text{dump}}} \left( \frac{\theta_{\text{dump}}}{\theta_{\text{pixel}}} \right)^{1/2} \approx \frac{\kappa \sqrt{2} T_{\text{sys}}}{\sqrt{\delta \nu}} \left( \frac{\nu_{\text{scan}}}{\theta_{mb}} \right)^{1/2}$
- Total time: $(\xi = 1/\sqrt{N_{\text{dump}}})$

$$t_{\text{OTF}} = 2\kappa^2 \eta_{\text{samp}} \eta_{\text{over}} \frac{L_x L_y}{\theta_{mb}^2} \frac{T_{\text{sys}}^2}{\sigma_{\text{pixel}}^2 \delta \nu} \frac{1}{N_{\text{pix}}} (1 + \xi)$$

- Ex: OTF-FSW, $10' \times 10', \kappa = 1, \xi = 0, \sigma_{\text{pixel}} = 0.5 \text{ K}, \eta_{\text{samp}} = 2.4, \eta_{\text{over}} = 1.2, T_{\text{sys}} = 500 \text{ K}, ^{12}\text{CO}(2 - 1): t_{\text{OTF}} = 2.6 \text{ hours}$
- OTF-PSW: $\kappa = 2, N_{\text{dump}} = 130, \xi \approx 1, \eta_{\text{over}} \approx 1.3: t_{\text{OTF}} = 5.7 \text{ hours}$
Observational Constraints

Some mapping techniques are better suited than others for your problem.

<table>
<thead>
<tr>
<th>method</th>
<th>line strength</th>
<th>linewidth</th>
<th>size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raster</td>
<td>any</td>
<td>any</td>
<td>small</td>
</tr>
<tr>
<td>OTF-PSW</td>
<td>strong</td>
<td>any</td>
<td>any</td>
</tr>
<tr>
<td>OTF-FSW</td>
<td>strong</td>
<td>narrow</td>
<td>any</td>
</tr>
</tbody>
</table>

Don’t forget about the Atmospheric Calibration!

- calibration: $t_{\text{CAL}} < 15$ min typically
  - move faster or shorten the row
- atmospheric cancelation: “the faster the best”
  - better repeat 10 times a map than doing it 10 times more slowly
Observational Limitations

Instrumental drifts $\rightarrow$ striping

$\lambda$ – scanning

$\beta$ – scanning

Repeat mapping in different directions (orthogonal is best) and average

Definitely better... but not optimum!
Imaging Recipes
Goal

- Sample the Sky Brightness distribution
- Reduce and Calibrate the data
- Produce a regularly sampled map of the Sky Brightness distribution

\[ T'_A(\theta, \phi) = \frac{1}{\Omega_A} \int_{\text{source}} P(\theta - \theta', \phi - \phi') J_{\nu}(T_B) \psi(\theta', \phi') d\Omega' \]
From a raw Sky map to a resampled grid
Sampling → multiplication  
Convolution → repetition  

Undersampling  
- loses the highest spatial frequency recoverable by the antenna  
- corrupts the highest frequencies of your data

In Fourier space, all information recovered if peak to peak separation is $> 2D/\lambda$

For telescope with diameter $D$  
$\text{HPBW} \approx 1.2 \frac{\lambda}{D} \text{ (taper)}$  
Highest spatial frequency : $D/\lambda$

$s(\theta) < 1/2 \frac{\lambda}{D} = \frac{1}{2} \text{HPBW}/1.2$ in Source space

Critical (Nyquist) frequency $f_c = 2D/\lambda$

$\theta_c = \frac{\text{HPBW}}{2.4}$
Sampling

Fig. 10.3  Demonstrating the sampling theorem.
CO 2–1 (230GHz) with HERA:
HPBW = 11.3” → \( \theta_c = 4.7” \)

If sampling \( \theta_s = \text{HPBW}/\eta \)
freq. \( f_1 = 1 / \theta_s \)

Lost/corrupted scales:
\( f_1 - f_c/2 < f < f_c/2 \)

HPBW/1.2 < \( \theta < \text{HPBW}/(\eta - 1.2) \)

\( \theta_s = 6” \): scales up to 15.6” are corrupted….
Convoluted and Imaging

\[ T_A'(\theta, \phi) = \frac{1}{\Omega_A} \int_{\text{source}} P(\theta - \theta', \phi - \phi') J_\nu(T_B) \psi(\theta', \phi') d\Omega' \]

**point-like source:**
- \( HPBW \gg \Omega_S \)

- \( P \otimes T_B = P(\Omega) \)

- \( \Rightarrow \text{see the beam} \)

**Infinite telescope:**
- \( HPBW \ll \Omega_S \)

- \( P \otimes T_B = T_B(\Omega) \)

- \( \Rightarrow \text{see the source} \)

**Resolve the source:**
- \( HPBW < \Omega_S \)

- \( P \otimes T_B \)

- \( \Rightarrow \text{smear the source} \)

**Source dimension must be deconvolved from the beam size**

If you measure \( l = 30'' \) and \( HPBW = 10'' \), the intrinsic source scale is

\[ l_0 = \sqrt{l^2 - HPBW^2} = 28.3'' \]
Summary

complex but not complicated
atmosphere: **better fast and repeat**
instrumental drifts: **ALWAYS two directions**
sampling: **full sampling,** $\theta_s < \theta_{mb}/2.4$
observing strategy: **optimized to ease data reduction**
start thinking in image and Fourier planes
proposal

1. justify the science
2. proposed observations (why do you absolutely need the 30m ?)
3. time estimate **must** allow to achieve scientific goals
Data Reduction
The Problem

Data reduction is a 3 step process:

Flux calibration (MIRA)

Bandpass calibration (CLASS90)

Imaging

Heterodyne mapping produces a huge amount of data

1deg² fully sampled map with HERA contains \( \approx 10^6 \) spectra: it is impossible to reduce individually each spectrum (1 month = 2.59x10^6 sec)

+ remember that faint features may be missed in individual spectra → data reduction is an iterative process in practice...

GILDAS package developed at IRAM has been designed to reduce spectral line data.
A typical data reduction session

Data Visualization and inspection
- Check spectra positions
- Inspection of data quality
- Check bandpass quality

Bandpass Calibration
- Baseline subtraction
- Data flagging

Map Making
- Convolution/resampling
- Data Cube exploration

Iterative Process in practice
For a reduced number of spectra: STAMP
Data Visualization

Rainfall (time) series of intensity vs velocity for 9000 spectra taken in 200 OTF scans
β- sorted map: velocity field looks more coherent:
You can now define windows to declare where the emission lies and subtract a baseline for each spectrum.
You may define several windows (interactively or not) depending on the number of velocity components.

Usually, a polynomial baseline of low order (from 0 to a few) is sufficient to correct for the drifts of the baseline.

In some cases (baseline ripple), it is possible to subtract a sine baseline function instead of a polynomial function, whose order would have to be rather high...
Map Making

- irregularly sampled spectra on the sky
  \[\rightarrow\] resample on a regularly spaced grid

- a two line operation for a two step process:
  - build a table: `table 'filename' new`
  - resample the data: `xy_map 'filename'`
Data Cube exploration

*Using the GILDAS task view*
Data Analysis
Data Analysis

distribution of the sky brightness distribution resampled on a regular grid
output: \textit{ppv data cube}
explore and extract information from a cube

See Talk by J. Pety on GILDAS
Data Analysis
Data Analysis
Position–Velocity Cuts

ppv: 3D

pv: 2D
Entrained gas (low-velocity)
High-velocity molecular gas component up to ~500 km/s!
Molecular Bullets beyond the optical jet
High-velocity gas emission between bullets?

Cernicharo & Reipurth (1996)
Examples on
How to extract some basic physical information from a line spectrum under reasonably wrong hypothesis
Radiative Transfer

- Basic Definitions
  The propagation of EM waves can be considered as straight lines (= rays) if $\lambda/D << 1$

  The specific intensity is defined by the following relation:

  $$dW = I_\nu \cos \Theta \ d\Omega \ ds \ d\nu$$

  $I_\nu$ = specific intensity (brightness) $Wm^{-2}Hz^{-1} \ sr^{-1}$
  $d\Omega$ = solid angle from which the radiation is coming
  $ds$ = element of surface
  $d\nu$ = element of bandwidth

  $I_\nu$ is the quantity collected by the telescope

  Total specific flux of the source:

  $$S_\nu = \int I_\nu \cos \Theta \ d\Omega \ [W \ m^{-2} \ Hz^{-1}]$$
Radiative Transfer

$I_v$ is conserved, independent of the distance, if there is no absorption process, and more generally, no interaction with matter along the line of sight.

Conservation of Energy implies that: $dW_1 = dW_2$

\[
I_{v,1} d\Omega_1 d\sigma_1 dv = I_{v,2} d\Omega_2 d\sigma_2 dv
\]

\[
d\Omega_1 = d\sigma_2 / R \quad \text{and} \quad d\Omega_2 = d\sigma_1 / R
\]

\[
I_{v,1} = I_{v,2}
\]
Radiative Transfer Equation

$I_\nu$ changes because of absorption/emission processes (interaction with matter). The variations of $I_\nu$ are described by the radiative transfer equation.

A simple phenomenological presentation: a slab of thickness $l$

\[
\text{Loss: } \frac{dI_\nu^-}{dl} = -\kappa_\nu I_\nu dl \\
\text{Gain: } \frac{dI_\nu^+}{dl} = \varepsilon_\nu dl \quad \text{(independent of } I_\nu\ldots)\
\]

\[
\frac{dI_\nu}{dl} = \varepsilon_\nu - \kappa_\nu I_\nu
\]

Case 1 (absorption only):

\[
I_\nu = I_\nu(0) \cdot e^{-\int \kappa_\nu I_\nu dl}
\]

Case 2 (emission only):

\[
I_\nu = I_\nu(0) + \int_0^S \varepsilon_\nu dl
\]
Radiative Transfer Equation

General case

Optical Depth $\tau_v : d\tau_v = \kappa_v dl$

Source function $S_v = \varepsilon_v / \kappa_v$

The radiative transfer equation now reads:

$$\frac{dI_v}{d\tau_v} = S_v - I_v$$

Formal Solution

$$I_v(s) = I_v(0) e^{-\tau_v(0)} + \int S_v(s') e^{-\tau_v(s')} \kappa_v(s') ds'$$

The emergent intensity is the sum of all the contributions along the path.
Local Thermodynamical Equilibrium

In a medium at LTE, the Kirchoff law holds:

\[ \frac{\varepsilon_v}{\kappa_v} = S_v = B_v(T) \]

where \( B_v(T) \) is the Planck function.

If the medium is isothermal: \( T(\tau) = \text{cst} = T \), the emerging intensity

\[ I_v(s) = I_v(0) e^{-\tau_v(0)} + \int S_v(s') e^{-\tau_v(s')} d\tau' \]

can be written as:

\[ I_v(s) = I_v(0) e^{-\tau_v(0)} + B_v(T) (1 - e^{-\tau_v(0)}) \]

At large optical depths \( \tau(0) >> 1 \) : \( I_v = B_v(T) \), independent of the nature of the radiating material...

- There are physical situations where the Source function \( S_v = \varepsilon_v/\kappa_v \) takes the form of a black body law.

- In the case of molecular line emission: the Kirchoff law can be applied if \( T \) is replaced by the excitation temperature \( T_{\text{ex}}^{ij} \) of the transition, which depends both on the upper and lower energy levels \( (i,j) \):

\[ \frac{n_j}{n_i} = g_j/g_i \exp \left( -\frac{(E_j - E_i)}{k T_{\text{ex}}^{ij}} \right) \]

\( n_i \) population of level \( i \)
\( g_i \) degeneracy of level \( i \)

\( B_v(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1} \)
Looking at the Sky

The Brightness Temperature is $T_B$:

$$I_v = B_v(T_B)$$

$$I_v(s) = I_v(0) e^{-\tau_v} + B_v(T) (1 - e^{-\tau_v})$$

Let $I_v(0) = B_v(T_0) = 2 kT_0 / \lambda^2$ (RJ-regime), it comes

$$T_B(v) = T \cdot (1 - e^{-\tau_v}) + T_0 \cdot e^{-\tau_v}$$

For spectral lines: only brightness above the cosmological background at 2.73K is detected →

$$T_B = (T - T_{bg}) (1 - e^{-\tau_v}) + T_0 e^{-\tau_v}$$
Molecular excitation

We consider a molecule whose first 2 Energy levels are populated.

\[
\begin{align*}
\text{u} & \quad E_u, \, n_u, \, g_u \\
\text{l} & \quad n_l
\end{align*}
\]

Two processes can populate the upper level:

- Collisions $\Rightarrow n_l \, n \, C_{lu} \, (T) \, \text{s}^{-1} \, (n = H_2 \, \text{density})$
- Radiation (photon absorption) $\Rightarrow n_l \, U \, B_{lu}$

Conversely,
- Spontaneous emission $A_{ul}$
- Stimulated Emission $B_{ul}$
- Collisions

\[
U = \text{mean radiation field} = 4 \pi / c \, I_v
\]

\[
\frac{dn_u}{dt} = n_l \, n \, C_{lu} + n_l \, U \, B_{lu} - n_u \, A_{ul} - n_u \cdot n \, C_{ul} - n_u \, U \, B_{ul}
\]

critical density $n_c : n_c = \frac{A_{ul}}{C_{ul}}$
CO as a probe of the Interstellar Gas

A study of HCL–2 in TMC1 (Cernicharo & Guelin 1987) from $^{12}$CO, $^{13}$CO, C$^{18}$O

Observing isotopomers with various opacities allow to probe different regions in the cloud.

CO is a privileged probe of molecular gas as it is:

- very abundant (~ $10^{-4}$ % H$_2$) except under « extreme » conditions (PDR, PSC)
- rather stable (even in shocks)
- $n_c$ ~ a few $10^2$ cm$^{-3}$, 1–0

$\rightarrow$ The lowest transitions are easily accessible and excitable:
- 1–0 : 115.2712 GHz $E_1 = 5.5$ K
- 2–1 : 230.53794 GHz $E_2 = 16.5$ K

So are its isotopomers

Typically: $[^{12}$CO]/$[^{13}$CO] = 60 and $[^{13}$CO]/[C$^{18}$O] = 8

$^{12}$CO is usually optically thick in molecular clouds...
CO observations of the HH111 core

Line intensities are expressed in $T_{mb}$:
Justified as the condensation is not very large.

Let’s assume the condensation is:
- homogeneous,
- at LTE $\rightarrow$ one single Tex (dense core n(H2) $>>$ nc)

For each transition, we can write:
$$T_B = (T_{ex} - T_{bg}) (1 - e^{-\tau v})$$

One has: $\tau^{12}/\tau^{18} = 480$ (abundance ratio)

If both lines are optically thin:
$$T_B^{12}/T_B^{18} = \tau^{12}/\tau^{18}$$

We measure $\sim 4 << 480$...
Hence the $^{12}$CO line is optically thick

$$T_B^{12}/T_B^{18} = 1 / (1 - \exp(-\tau^{18})) : \tau^{18} = 0.3,$$
and $\tau^{12} \sim 140$...
Temperature in HH111

12CO is optically thick, its emission is resolved, then:

\[ T_{B12} = (T_{ex} - T_{bg}) \approx T_K - 2.7 \rightarrow T_K \approx 20 \text{K in the core} \] (outer layers)

We can also estimate \( T_K \) deeper in the core from the \( \text{C}^{18}\text{O} \) lines, which are both optically thin.

One more assumption here: both transitions have same \( T_{ex} \).

\[ T_{21} = 4 \text{K} \]
\[ T_{10} = 5 \text{K} \]

\[ R_2 = 4 \exp \left( -\frac{h\nu_{12}}{kT_{12}} \right). \]

\[ R_2 \rightarrow T_{12} = 30 \text{K} > 20 \text{K} \]

Is there a heating source inside the core or…

Are we measuring comparable things?

Not completely:
- the LTE approx. may not be valid, after all?
- the beam does not probe the same region at 3 and 1.3mm

\[ \rightarrow \text{First check the source size wrt beam size to conclude} \]
It is possible to derive the following relation assuming LTE conditions are fulfilled.

\[
N_{\text{CO}} = \frac{3ch \int \tau_v dv}{8\pi^3 \nu_{ij} |\mu_{ij}|^2 \left[1 - \exp \left(- \frac{h\nu_{ij}}{kT_{ij}} \right) \right] f_i},
\]

\(\mu_{ij}\) is the dipole moment of the rotational transition. For a linear molecule, it reads:

\[
|\mu_J|^2 = \mu_0^2 \cdot \frac{J+1}{2J+3}, \quad J+1 \rightarrow J
\]

For CO (and its isotopomers): \(\mu_0 = 0.112\) D

You have:

\[
N(\text{CO}) = 1.06 \times 10^{13} \ T_{21} \ \exp\left(16.5/T_{21}\right) \ \int \ T_R(2-1) dv \ \text{cm}^{-2}.
\]

\(V\) in km/s

You can do the same for C18O:

...Good Luck!
That’s All!

Thanks to the IRAM staff to have made the 30m such a reference in the world of radioastronomy, especially to Albert Greve, Juan Penalver, Carsten Kramer and Clemens Thum, not only for all the slides I took from their presentations but for all their papers and work on the calibration of this outstanding instrument.

The calibration manual of the IRAM 30m should be read by any radioastronomer (just like the paper by Kutner and Ulich!).

http://www.iram.es/IRAMES/otherDocuments/manuals/Report/cali_rep_ddo970205.ps